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THE RANGE AND RANGE RATE SYSTEM AND DATA ANALYSIS FOR SYNCOM I (1963 4A)

by H. W. Shaffer, W. D. Kahn, W. J. Bodin, Jr., G. C. Kronmiller, P. D. Engels, and E. J. Habib

Goddard Space Flight Center
Greenbelt, Maryland

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SUMMARY

A range and range rate system similar to the Goddard Range and Range Rate System was used to track the Syncom I through the transfer ellipse until the apogee kick motor fired. This report discusses the Syncom Range and Range Rate System operation and the data processing. The standard deviations achieved were 15.49 m in range and 0.05 m/sec in range rate with respect to the calculated orbital elements. Analysis of the data over short continuous intervals shows the data to have an accuracy better than 20 m in range, with either the 100 or 20 kc ranging tone, and a range rate accuracy within 0.05 m/sec.

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INTRODUCTION

The Syncom Range and Range Rate System, which is similar to the Goddard Range and Range Rate System (Reference 1), was developed to track the Syncom I satellite, launched on February 14, 1963, during the transfer ellipse and while it was in the synchronous orbit. Equipment was developed which utilized the existing communications transmitter, 30 foot parabolic antenna, spacecraft communication transponder, and ground preamplifier (Figure 1). Because of weight limitations, a range and range rate transponder could not be placed on the spacecraft, therefore all measurements had to be made with the spacecraft's communication transponder. This required the up link frequencies to be 7361.275 Mc for channel I and 7363.00 Mc for channel II. The down link frequencies were 1814.069 Mc for channel I and 1815.794 Mc for channel II. Also, a beacon frequency of 1820.177 Mc was transmitted from the spacecraft.

The two field installations for tracking were Lakehurst, New Jersey, for the synchronous orbit, and a ship located in the harbor of Lagos, Nigeria, for the transfer ellipse, the injection into the synchronous orbit, and the synchronous orbit. Data were to be taken during two 15 minute segments, one when the satellite was between 10,000 and 15,000 km, the other when the satellite was between 30,000 and 35,000 km. The data in this report were acquired by the tracking ship, but only during the transfer ellipse (because of the failure of spacecraft communications after the apogee motor fired).

This report presents the computed standard deviations with respect to the calculated orbital elements and an analysis of the data for continuous functions of time over short intervals.

DESCRIPTION OF THE SYNCOM RANGE AND RANGE RATE SYSTEM

Range measurements are accomplished by utilizing the principle that any wave or group of waves propagated at a given velocity experiences a delay which depends upon the distance traversed. Range rate measurements utilize the principle that a wave emanating from an object

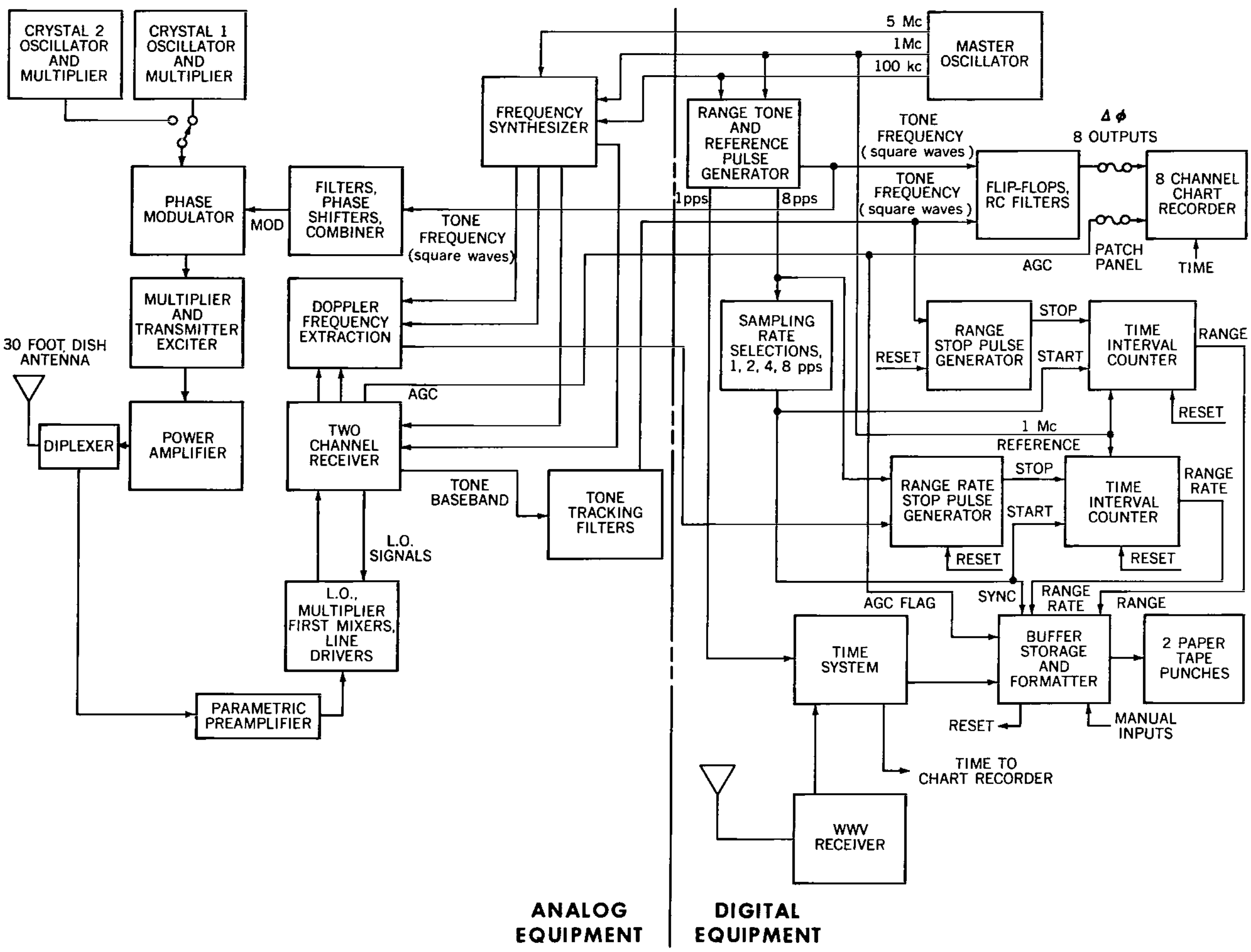

Figure 1—Block diagram of the Syncom Range and Range Rate System.

which is in motion relative to an observer arrives at the observer at a different frequency than that at which it emanated.

Figure 1 shows the block diagram of the Syncom Range and Range Rate System. The tracking station generates a carrier plus sideband pairs obtained by phase modulating the carrier with the ranging tones of 100, 20, 4.800, 4.160, 4.032, 4.008, and 4 kc (Figure 2). The 800, 160, 32, and 8 cps range tones are complemented on the high side of 4 kc to assure that the carrier will be absolutely free of modulation products for range rate or Doppler measurements.

A reference data rate of 8, 4, 2, or 1 cps, synchronized to WWV, initates the counting of 10 Mc by the time interval units, which measure the phase delay of the ranging tones and the time necessary for counting 81924 cycles of Doppler frequency plus bias frequency.

The 100 kc sidetone phase delay to the spacecraft and return is measured to an accuracy of 1 percent by controlling the 100 kc phase-locked loop so that the deviation of the phase of the 100 kc signal and the phase-locked loop frequency is less than 3.6 degrees. This equals 1 count

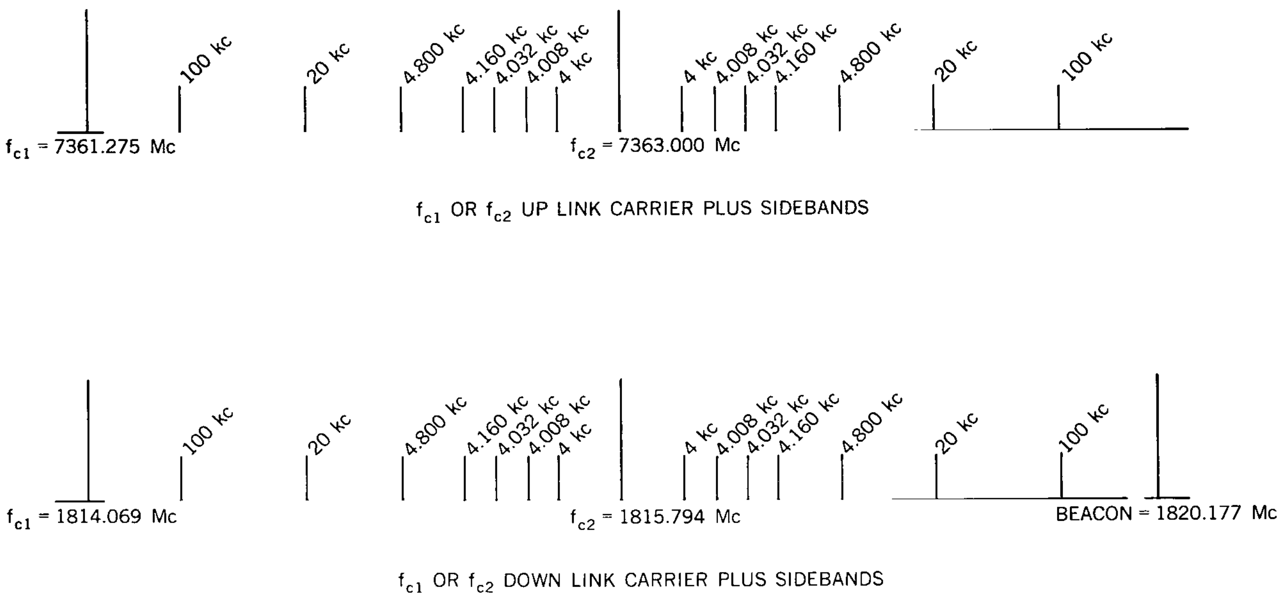

Figure 2—Spectra.

of 10 Mc which in turn is equivalent to a resolution of ±15 meters. The lower range tones of 20 kc, 4 kc, 800 cps, 160 cps, 32 cps, and 8 cps are controlled so that their individual phases do not vary more than ±36 degrees, which resolves the ambiguity of the next higher tone to the whole cycle.

The carrier of the ground transmitter is used in the mixer operations for the range rate measurements. It is derived from one of two ultra-stable oscillators, the one used depending on frequency selection. The carrier and the ranging sidetones are received at the spacecraft transponder, which acts as a booster amplifier and translates the spectrum down to either 1814.069 or 1815.794 Mc from 7361.275 or 7363.000 Mc, respectively (Figures 2 and 3).

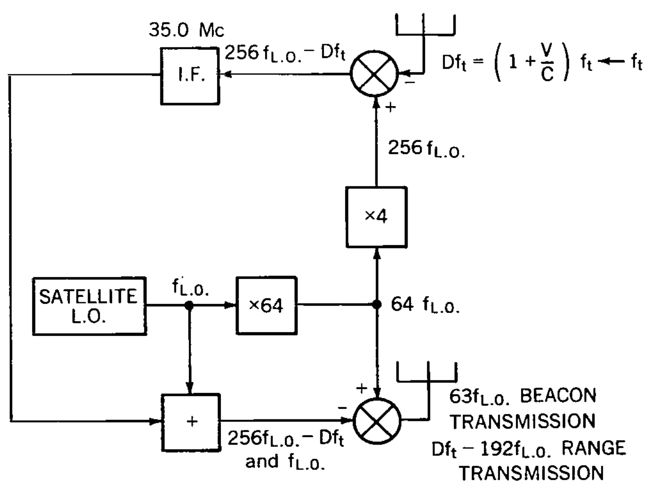

Figure 3—Spacecraft transponder.

The receivers at the tracking station receive the transmitted carrier frequency from the ground transmitter minus 192 times the satellite local oscillator frequency for the range tone frequency. For the beacon frequency they receive 63 times the satellite local oscillator frequency. The receivers provide initial mixing and phase locking to the transmitted carrier frequency and beacon frequency. This produces a difference frequency equal to the sum of the following:

1. The satellite local oscillator frequency.
2. The one-way Doppler shift of the carrier in the 7.3 Gc range.
3. The one-way Doppler shift of the ground-transmitted carrier frequency minus 192 times the satellite local oscillator frequency.
4. The one-way Doppler shift of 63 times the satellite local oscillator frequency.
5. The satellite local oscillator drift.
6. The Doppler frequency (Figure 4).

The ranging frequency (ground-transmitted carrier frequency minus 192 times the satellite local oscillator frequency) is converted by utilizing phase-locked oscillators for carrier and range tone detection.

The Doppler extractor mixes the outputs of both receivers to remove all satellite local oscillator drifts and produces the two-way Doppler shift of the transmitted carrier impressed on a bias frequency of 1 Mc (Figure 4). The range tones from the ranging receiver are actually demodulated in the range extractor, which uses narrow-band phase-locked loops for separating and improving the signal-to-noise ratios of the individual range tones. The 1-Mc-biased Doppler is fed to a digital unit of the Doppler extractor which counts at a 10 Mc rate in a second time interval unit for a period of 81924 counts of the 1-Mc-biased frequency ± the Doppler frequency. Ranging data may be extracted at the rate of 8, 4, 2, or 1 time per second. The 8, 4, 2, and 1 cps are synchronized

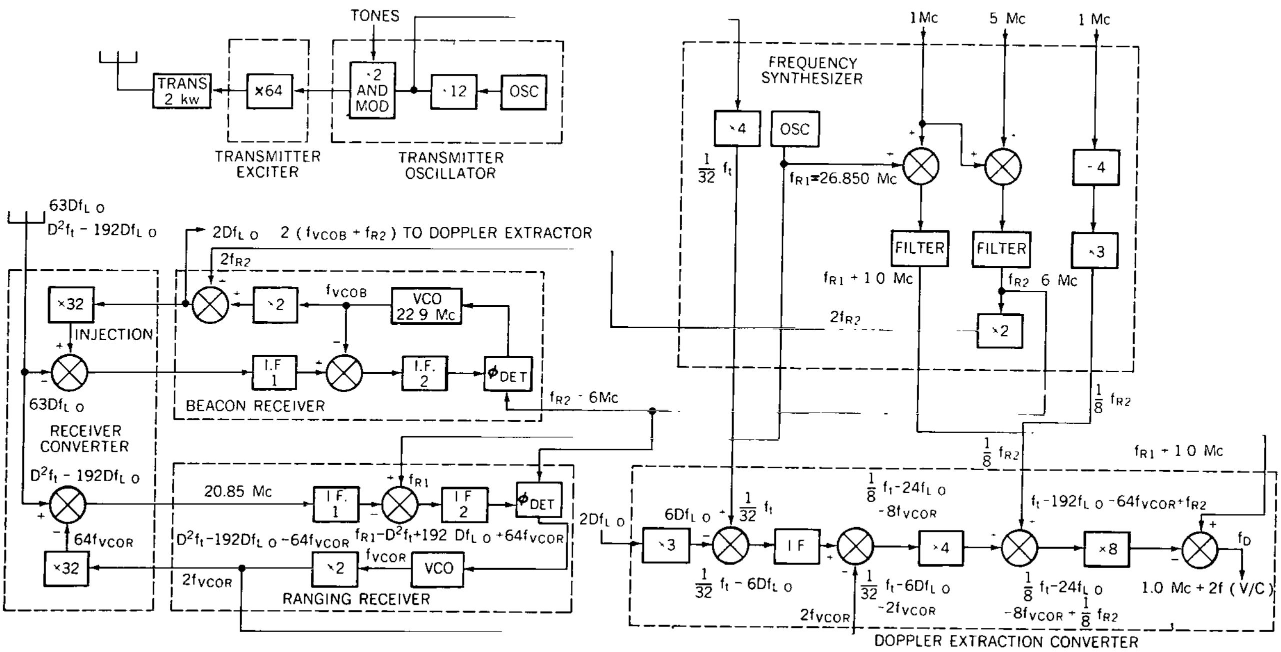

Figure 4—Receivers and doppler extraction process (f_{VCOR} is the frequency of the VCO in the ranging receiver and f_{VCOB} is the frequency of the VCO in the beacon receiver).

to WWV and used as start pulses to start the 10 Mc range time interval unit counting. During calibration the tones are aligned alternately positive and negative with the 100 kc tone going positive. All ranging tones have the same phasing characteristics at 8, 4, 2, and 1 times per second. The time interval between the start and stop pulses, with respect to the speed of light, is the range to and from the spacecraft.

The digital outputs from both range and range rate (Doppler) time interval units are multiplexed with the output of the antenna azimuth and elevation and the output of the digital clock. They are punched on two separate teletype punch units which produce data at the reference rate in teletype code for transmission to Goddard Space Flight Center.

Functions of Subsystems

Master Oscillator

This oscillator, with its associated distribution amplifier and stand-by power supply, provides a stable frequency source of 5 parts in 10^{-10} per second and 5 parts in 10^{-10} per day at 5 Mc, 1 Mc, and 100 kc for the frequency synthesizer, time interval counters, digital clock, and range tone and reference pulse generator.

Range Tone and Reference Pulse Generator

The range tone and reference pulse generator produces, through a digital countdown from the 1 Mc of the master oscillator, eight harmonically related sidetones: 500 kc, 100 kc, 20 kc, 4 kc, 800 cps, 160 cps, 32 cps, and 8 cps. The countdown is continued from 8 cps for the 8, 4, 2, and 1 cps reference pulses or data rates. The countdown unit is clocked so that as much jitter as possible may be eliminated.

Filters, Phase Shifters, and Combiner

The countdown outputs for the range tones are square waves, therefore requiring low Q filters to provide sine waves that can be used in the electromechanical phase shifters. The tones are fed to individual phase shifters which adjust their phase and remove any differential phase delay throughout the system, including the transmitter, spacecraft communication transponder, and receivers. The individual phase shifters have a mechanical linkage ratio equal to the transmitter range tone ratio. The combiner simply adds all range tones to form a single complex wave for phase modulating the carrier frequency.

Transmitting Section

The transmitting section consists of two basic crystal oscillators at different frequencies to derive the two transmitting frequencies. The two frequencies correspond to the two channels of the satellite transponder, and the channel selection is a ground function of choosing one of the two crystal oscillators, the nominal being 4.79 Mc. The chosen oscillator output is processed through several multipliers which multiply the basic frequency by 12 to approximately 57.6 Mc. This is used in the frequency synthesizers (for Doppler extraction) and is also used as the input to the phase modulator. It is phase modulated by the complex wave representing the summation of the ranging tones in the phase modulator. The phase-modulated 57.6 Mc is multiplied by 2 and sent to the transmitter exciter which multiplies the 115.2 Mc to approximately 7.3 Gc. Then the 7.3 Gc is amplified to 20 kw and transmitted to the satellite.

Beacon Receiver (Doppler)

The beacon receiver operates in either search or track mode. During the search mode the voltage-controlled oscillator (VCO) is not in the locked condition and is operating at a crystal-controlled frequency of approximately 22.9 Mc. The VCO frequency is repeatedly scanned through a frequency spectrum which includes the satellite beacon transmission signal. In the locked mode the VCO follows the phase of the input signal.

Twice the VCO frequency is heterodyned with twice the reference frequency, f_{R2} (= 6 Mc). In the receiver converter the thirty-second harmonic of the above mixer is heterodyned with the satellite beacon transmission frequency (Figure 4).

The output of IF amplifier 2 is 6 Mc ± the Doppler frequency; this is fed into the loop phase detector which compares this frequency with f_{R2} (6 Mc). If the signals are not of the same frequency, the output of the phase detector will be a sinusoidal error signal, with a beat frequency equal to the frequency difference of the two inputs. If the signals are of the same frequency, which occurs first in acquisition, the output then will be a dc error voltage equal in magnitude and polarity to the phase difference of the two input signals. The output of the phase detector controls the frequency and phase of the VCO so that they equal the frequency and phase of the input signal. The VCO is a crystal-controlled oscillator which has a normal resting frequency of approximately 22.9 Mc, but the frequency and phase of the VCO are controlled by a dc voltage. The output of the VCO multiplied by 2 + 2 f_{R2} (2 f_{R2} = 12 Mc) is used in the Doppler extractor.

Both receivers have a threshold of -154 dbm.

Ranging Receiver

The operation of the ranging receiver is similar to that of the beacon receiver. The sixty-fourth harmonic of the VCO frequency is heterodyned with the satellite-range-transmitted frequency, and the difference, 20.85 Mc, is fed to the first IF. The 20.85 Mc is then mixed with f_{R1} (26.85 Mc) from the frequency synthesizer; this produces a 6 Mc ± Doppler frequency output. One of the outputs of the second IF is fed to a modulation phase detector which extracts the range tones. The other output is fed to the VCO.

During the search mode the AGC, which is in the ranging receiver only, is not activated. During the locked mode, the same operations as those of the beacon receiver are performed, the VCO follows the phase of the input signal, and the AGC is in operation.

Doppler Extractor

The Doppler extractor consists of mixers, multipliers, and amplifiers. The beacon frequency, $2\ Df_{L.o.}$ (D is the Doppler-shifted frequency), is multiplied by 3 in the first multiplier (Figure 4) to produce $6\ Df_{L.o.}$, which is mixed with $(1/32)\ f_t$ to yield $(1/32)\ f_t - 6\ Df_{L.o.}$. The output is mixed with $2\ f_{VCOR}$ (f_{VCOR} is the frequency of the VCO in the ranging receiver) to produce $(1/32)\ f_t - 6\ Df_{L.o.} - 2\ f_{VCOR}$ which is then multiplied by 4 to give the output at the times 4 multiplier of $(1/8)\ f_t - 24\ f_{L.o.} - 8\ f_{VCOR}$. This is mixed with $(1/8)\ f_{R2}$ (750 kc) to produce $(1/8)\ f_t - 24\ f_{L.o.} - 8\ f_{VCOR} + 1/8\ f_{R2}$. This is subtracted from f_{R1} + 1 Mc $\left(f_{R1} = 27.85\ \text{Mc}\right)$ which is equal to $1.0\ \text{Mc} \pm 2\ f\ \left[1 \pm (v/c)\right]$.

The $1.0\ \text{Mc} \pm 2\ f_D$ is counted by an "N" counter which counts a set number of 81924, synchronized with the data rate. The second pulse of the 81924 pulses starts the counting of the range rate time interval unit at 10 Mc. The pulse following the 81924 count, pulse 81925, stops the time interval unit. This allows the interval to vary about 819240 microseconds, depending upon the magnitude of the Doppler and whether it is positive or negative Doppler. The time interval is multiplexed with other data and punched on paper tape.

Range Extractor

One of the outputs of the second IF in the ranging receiver is fed to a modulation phase detector which feeds the range tones to the high and low range tone filters (Figure 5).

High Range Tone Filter

The input portion of the high range tone filter contains a predetection band separation filter. This filter consists of 100 and 20 kc band-pass amplifiers, and a 10 kc low pass RC filter and amplifier. The 500 kc band-pass amplifier output is connected directly to the tone transfer relay. The 100 and 20 kc band-pass amplifier outputs are connected to limiters which maintain the signal-to-noise power at a constant level. The outputs of the limiters are connected to the tone transfer relays. The 10 kc low pass amplifier delivers signals to a 4 kc band-pass filter and to the low range tone filter. The output from the 4 kc band-pass filter is connected to the 4 kc tone transfer relay.

The reference tones input to the high range tone filter are connected to the tone transfer relay via the reference tones input gate. The reference tones are always present until the signal is detected. This enables the phase lock loops to rest at the exact range tone frequency transmitted, and the loop has to shift by the amount of Doppler that is present on the signals.

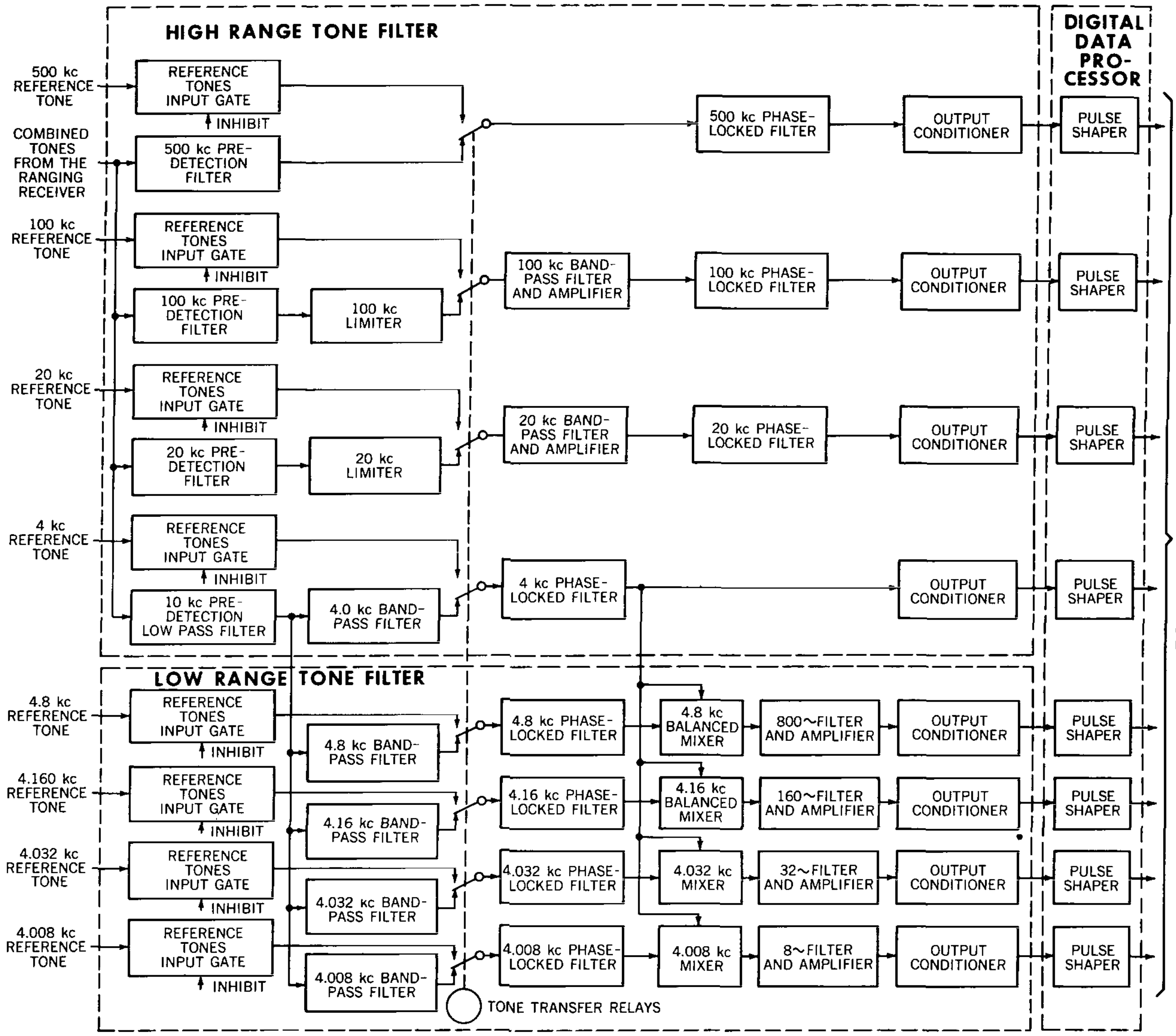

Figure 5—High and low range tone filters.

The 100 and 20 kc channels contain additional band-pass amplifiers to remove the harmonic components generated by the limiters. The phase-locked filter output signals are accurate reproductions of the input signals except that they are shifted exactly 90 degrees in phase and the noise and distortion are removed. The resulting sine waves from the high range phase-locked filters are converted to pulses and fed to a stop pulse generator. The pulse shaper output signals are pulses with maximum rise times of less than 10 nanoseconds occurring at the same rate as the input tones.

Low Range Tone Filter

The input to the low range tone filter is received from the 10 kc low pass predetection filter in the high range tone filter. The output signal from the predetection filter is connected to the

4.8, 4.16, 4.032, and 4.008 kc band-pass amplifier stages. These filters separate the individual tones from the composite signal. The outputs of the filters are connected to one side of the tone transfer relays and the reference tones are connected to the other. The relays perform the same function as described for the high range tone filter.

The outputs of the relays are fed to the phase-locked loops. The output signals from each phase-locked loop are fed to a balanced mixer. The second mixer input is the output of the 4 kc phase-locked loop and the mixer output is the difference frequency. The outputs of the mixers are passed through filter amplifiers only for more filtering effect. The outputs of the filters are converted to pulses and fed to a stop pulse generator.

Range Stop Pulse Generator

The negative going pulses, corresponding to the negative going zero crossings of the delayed range tones from the high and low range tone filters, are individually fed to the set inputs of individual flip-flops (Figure 6). The 8 cps return pulse sets the 8 cps flip-flop which in turn enables the 32 cps flip-flop. In a similar manner each subsequent flip-flop is enabled by the set condition of the previous flip-flop. Since the phase relation of the 8 tones had previously been adjusted during calibration, the 8 flip-flops are reset in sequence from the lowest tone chosen. When the

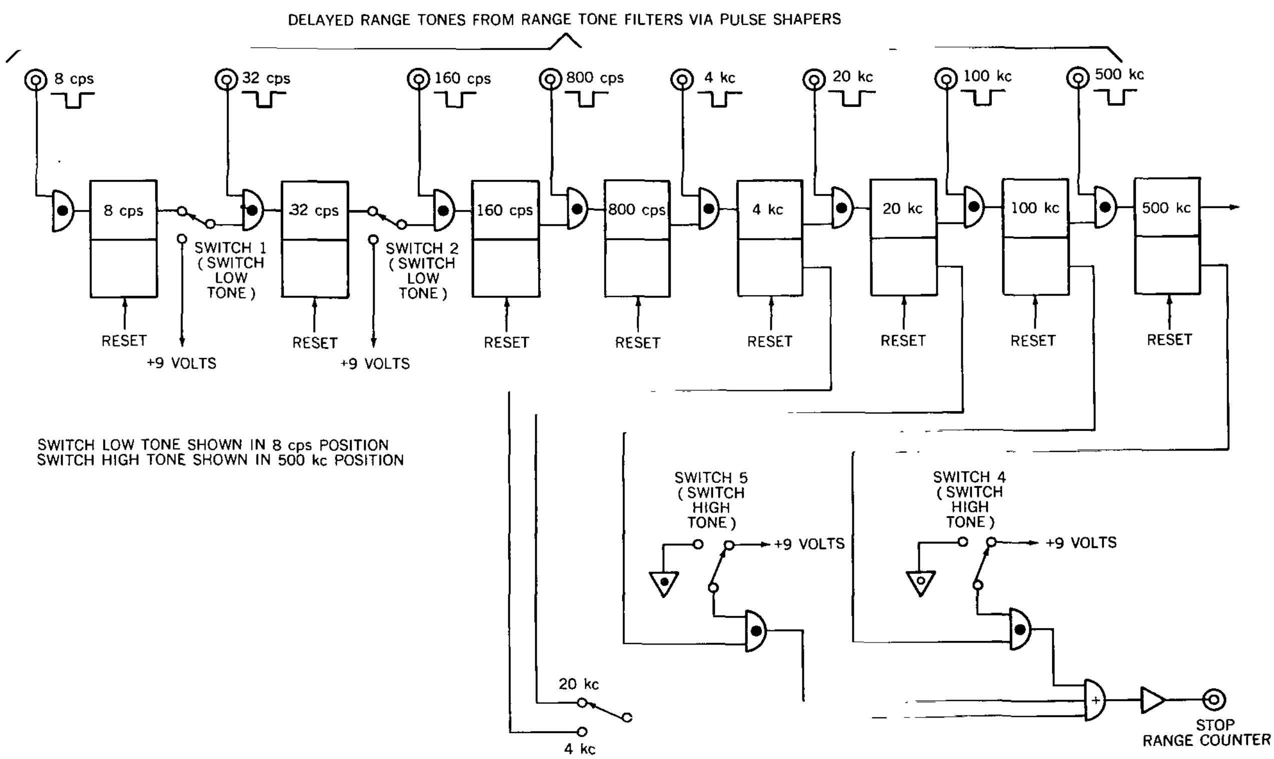

Figure 6—Range stop pulse generator.

highest tone chosen has reset its respective flip-flop, a pulse is generated which is used as the stop pulse for the time interval unit.

A choice of tones is available. Either 8, 32, or 160 cps is available for the lowest tone used and the choice is usually dependent on the altitude of the spacecraft; 100, 20, or 4 kc may be chosen for the higher sidetones, by eliminating frequencies from the highest, in sequence.

Satellite Communication Transponder

The range and range rate measurements are made by utilizing the satellite communication transponder (Figure 3), which has a wide bandwidth and essentially no time delay for the ranging tones. It is a booster amplifier and shifts the spectrum from 7.3 to 1.8 Gc. Fundamentally, range and range rate measurements were made with zero added weight and used very little spacecraft power.

Antenna

The Syncom tracking stations at Lakehurst, New Jersey, and on the U. S. N. S. Kingsport (tracking ship) utilize a 30 foot parabolic antenna for transmitting and receiving. The antenna uses the conical scan principle and has approximately a 1.8 degree beamwidth at 1.8 Gc. The data output of azimuth and elevation has an accuracy of 0.1 degree.

Parametric and Power Amplifiers

The parametric amplifier is a 3 stage device which has 40 db gain and a noise figure of approximately 2.7 db.

The Syncom Range and Range Rate System supplies the actual transmitting frequency to the power amplifier which serves to amplify the signal to 20 kw for transmission.

Calibration of the System

Calibration eliminates any differential phase delay in the system and any bias errors which may be present. The system is operated as a closed loop through a transponder at a known distance from the antenna. After the receiver and range tone phase-locked loops are locked to the signal, the individual range tones are set positive and negative, alternately with each other, by the phase resolvers. All the resolvers are then rotated simultaneously, which in effect rotates the stop pulse, until the range time interval unit reads the time required for the range tones to traverse the known distance to and from the calibration transponder.

The range, range rate, antenna pointing angle, time, and station data are multiplexed and punched on paper tape for transmission to Goddard Space Flight Center.

DATA DESCRIPTION AND HANDLING

Syncom I was launched from Cape Canaveral, Florida, on February 14, 1963. The transfer ellipse phase of the trajectory was south of Lagos, Nigeria, and southeast of Maligasy. After this phase the altitude was approximately 35,000 km. A rocket motor was fired at apogee to place the satellite into a synchronous orbit.

The tracking data presented in this report were acquired by the tracking ship in the harbor of Lagos, Nigeria. They consist of two 15 minute segments, separated by approximately 2 hours and 19 minutes. The first segment was acquired when the satellite height was between 10,000 and 15,000 km. The highest range tone used for making range measurements was 100 kc.

During the second segment of data, the satellite altitude above the earth was between 30,000 and 35,000 km. For the first 5 minutes of this segment the 4 kc tone was used, because of communication problems. The 100 and 20 kc tones were turned off. This provided more power in the lower tones in order to receive ranging data; though the range accuracy was degraded to ±500 m with the 4 kc tone. The power in the 4 kc tone could not be increased to decrease the phase jitter which is due to the 4.008 kc phase-locked loop locking on the 4 kc tone. After approximately 5 minutes, the 20 kc tone was turned on and the modulation index increased, increasing the power in the 20 kc tone, which decreased the phase jitter of this tone. This tone was used as the highest ranging tone for the rest of this interval. The deviation in range with the 20 kc tone was less than ±20 m.

During the two tracking intervals 1052 measurements of range data and 945 measurements of range rate data were recorded. At the Goddard Space Flight Center the 1052 points of range data, including the 4 kc data, were smoothed to 68 points of which 55 points were used for final orbital calculations. If the measurements using the 4 kc tone are excluded, only 5 smoothed points were eliminated from the calculations. The 945 points of range rate data received were smoothed to 71 points and all 71 were used in the final orbital calculations.

The data from the system is punched on paper tape for transmission to the Goddard Space Flight Center in the format shown in Figure 7. The received data is processed in a two-computer operation. The reproduced teletype tape with the data is first processed in the CDC-160 computer which performs the following operations:

1. Computes range in meters from the time of propagation or phase delay of the ranging tones to the spacecraft and return.

2. Computes range rate in m/sec from the time interval required to count "N" cycles of a standard frequency ± the Doppler frequency of the carrier.

3. Reproduces the 30 foot communications antenna angle readout which the system measures to 0.1 degree.

4. Makes the necessary adjustments to time for the propagation time of the ranging tones to and from the spacecraft, and for the difference in time between the sampling of the time register and the range and range rate registers.

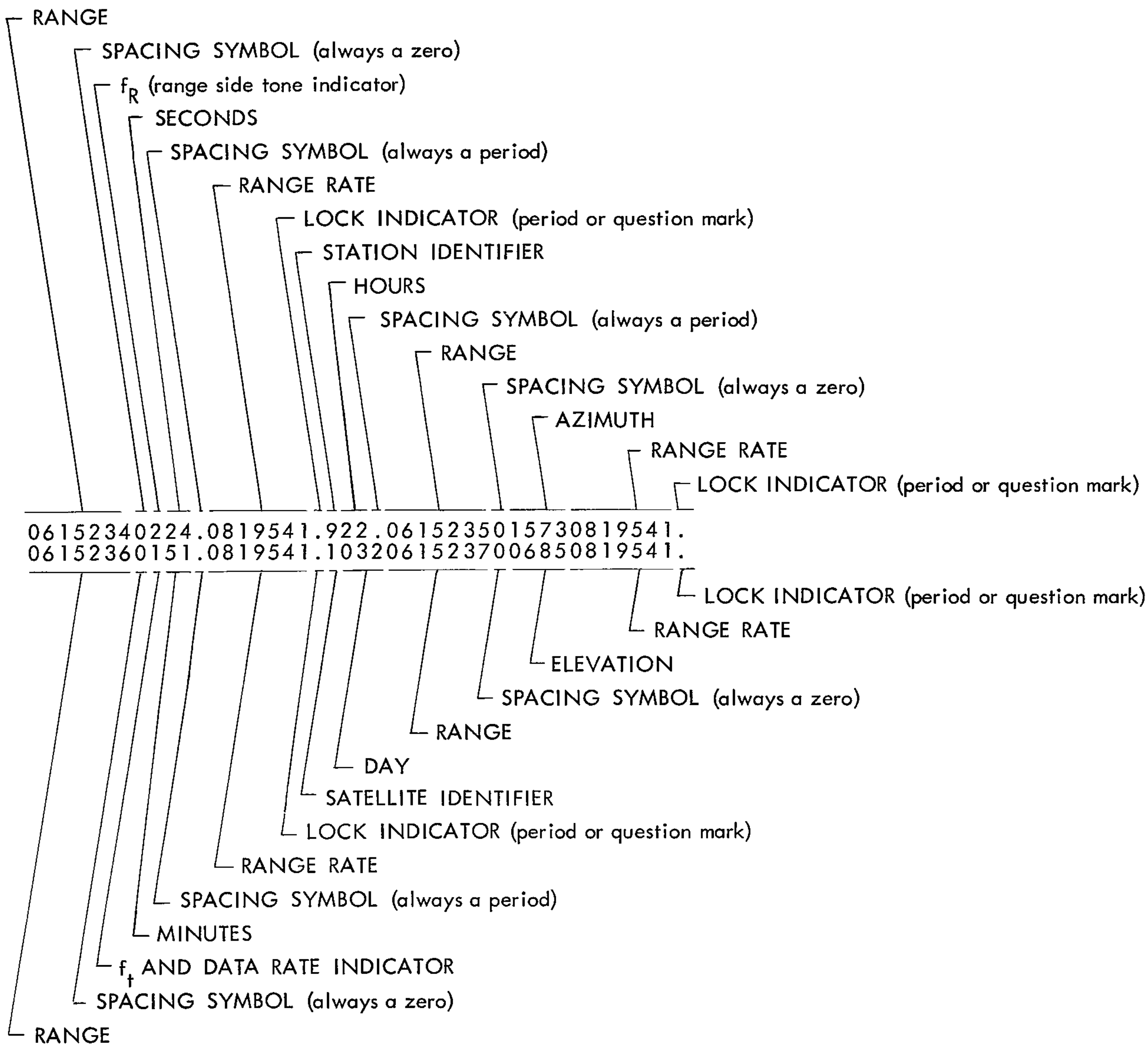

Figure 7—Data format.

The raw data from the CDC-160 is reproduced on punched cards and magnetic tape for processing by the IBM 7090 or 7094 computer, which uses the punched cards and magnetic tape for an input program. The data is then smoothed by fitting a quadratic or higher degree polynomial to it in the least squares sense. After smoothing, a differential correction program is applied to the data by using either the Brouwer or MCOI orbit generators for calculating the orbital elements and standard deviations.

DATA ANALYSIS

Computer Results for Standard Deviations

The MCOI orbit generator was used for the Syncom I satellite. It is a numerical integration program that applies a Runge-Kutta Gill Integration Technique (Reference 2) to the solution of the equations of motion, by assuming a spheroidal earth. The integrated equations of motion include effects due to the principal term and second, third, fourth, and fifth harmonics of the earth's potential, as well as lunar and solar perturbations, drag, and solar radiation. For lunar and solar perturbations the program requires as input, ephemerides' tapes for the sun and moon. Drag effects are computed by assuming an exponential atmosphere with a constant temperature gradient. The standard output is an ephemeris tape giving the position and velocity vectors of the satellite at a given interval of time. The differential correction program uses the position and velocity vectors to calculate the orbital elements and their standard deviations. The standard deviation for Syncom I was 15.49 m in range and 0.05 m/sec in range rate.

The calculated orbital elements used range and range rate data and angular data from the Johannesburg Minitrack station. The range data had a weighting factor of 40, the range rate data had a weighting factor of 24, and the Minitrack data had a weighting factor of 1. Therefore the orbital elements were heavily influenced by the range and range rate data.

The Minitrack data agreed with the calculated orbital elements to 0.12 milliradian or about 25 seconds of arc, which proves that the range, range rate, and Minitrack data agree with the calculated orbital elements.

Analysis of Range and Range Rate Data Over Short Continuous Intervals

For analysis of the range and range rate data over short continuous intervals a polynomial fit was made to the data. It was found that a second degree polynomial best fit the data in all the cases considered. That is

$$g_k = a_0 + a_1 k + a_2 k^2 , \tag{1}$$

where

$g_k = g(t_k)$, the value of the function at time t_k, the function being either range or range rate,

$k = \dfrac{t_k - t_o}{\Delta t} \qquad (k = 0, 1, 2, \cdots, n-1)$;

t_k = the time corresponding to the k^{th} observation,

t_o = the time corresponding to the initial observation,

Δt = the time interval between successive observations.

The total interval over which the data is time continuous is defined by

$$T = t_{n-1} - t_o \ .$$

Application of the method of least squares (Reference 3) gives the coefficients (a_0, a_1, a_2) and the standard deviation of fit of the polynomial to the data. That is, the normal equations are:

$$\begin{bmatrix} \sum_{k=0}^{n-1} g_k \\ \sum_{k=0}^{n-1} kg_k \\ \sum_{k=0}^{n-1} k^2 g_k \end{bmatrix}_{(3\times1)} = \begin{bmatrix} n & \sum_{k=0}^{n-1} k & \sum_{k=0}^{n-1} k^2 \\ \sum_{k=0}^{n-1} k & \sum_{k=0}^{n=1} k^2 & \sum_{k=0}^{n-1} k^3 \\ \sum_{k=0}^{n-1} k^2 & \sum_{k=0}^{n-1} k^3 & \sum_{k=0}^{n-1} k^4 \end{bmatrix}_{(3\times3)} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}_{(3\times1)} \tag{2}$$

In abbreviated matrix form Equation 1 is:

$$G_{(k\times1)} = T_{(k\times3)} A_{(3\times1)} \ , \tag{3}$$

where $k > 3$. The resultant normal equations given by Equation 2, in abbreviated form, are

$$\left[T^T_{(3\times k)} G_{(k\times1)}\right]_{(3\times1)} = \left[T^T_{(3\times k)} T_{(k\times3)}\right]_{(3\times3)} A_{(3\times1)} \ . \tag{4}$$

The solution of Equation 4 is given by

$$A_{(3\times1)} = \left[T^T T\right]^{-1}_{(3\times3)} \left[T^T G\right]_{(3\times1)} \ . \tag{5}$$

From this least squares solution the measure of how well the data fits the assumed polynomial is given by the value of σ. The meaning of σ is that the probability will be 68.3 percent that all the data fitted to the polynomial will lie in the region bounded by $+\sigma$ and $-\sigma$.

For evaluating σ, use is made of the solution to Equation 2 or 4 as follows:

$$\sigma = \pm \frac{1}{n-3} \left[\sum_{k=0}^{n-1} g_k^2 - a_0 \sum_{k=0}^{n-1} g_k - a_1 \sum_{k=0}^{n-1} kg_k - a_2 \sum_{k=0}^{n-1} g_k k^2 \right]^{1/2} \ , \tag{6}$$

where $n > 3$. In matrix form

$$\sigma = \pm \left\{ \frac{1}{n-3} \left[\left(G^T G\right) - A^T \left(T^T G\right) \right] \right\}^{1/2} \ . \tag{7}$$

And it is from this point of view that the analysis of range and range rate data is considered here.

Figures 8 and 9 are plots of the residuals in the region where the satellite height was between 10,000 and 15,000 km. The highest range tone used in this region was 100 kc. The residuals represent deviations of the raw data from the second degree polynomial fitted to the data in the least squares sense.

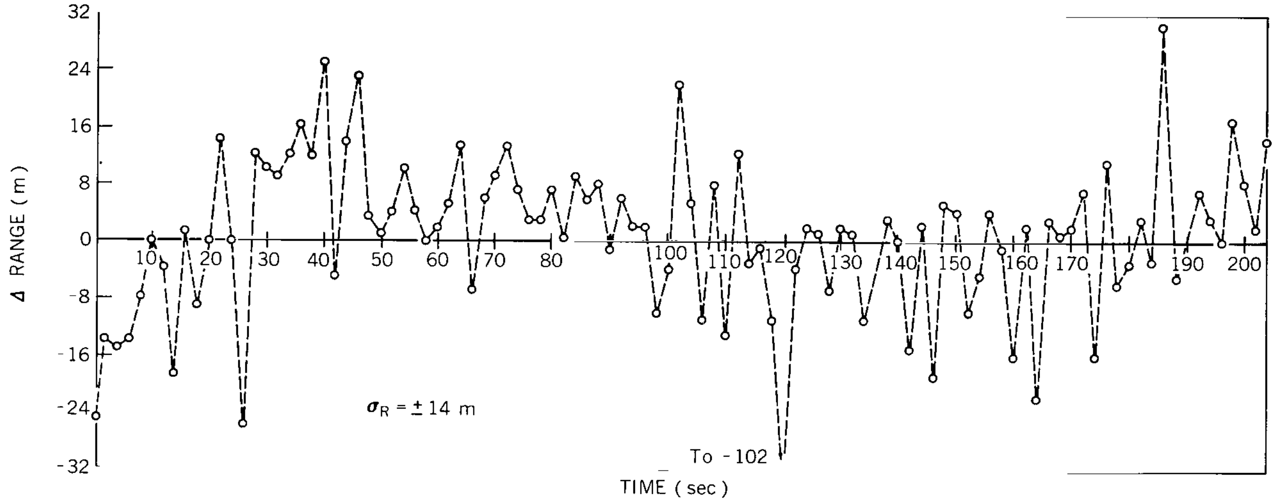

Figure 8—Data at 10,000 - 15,000 km (100 kc tone range).

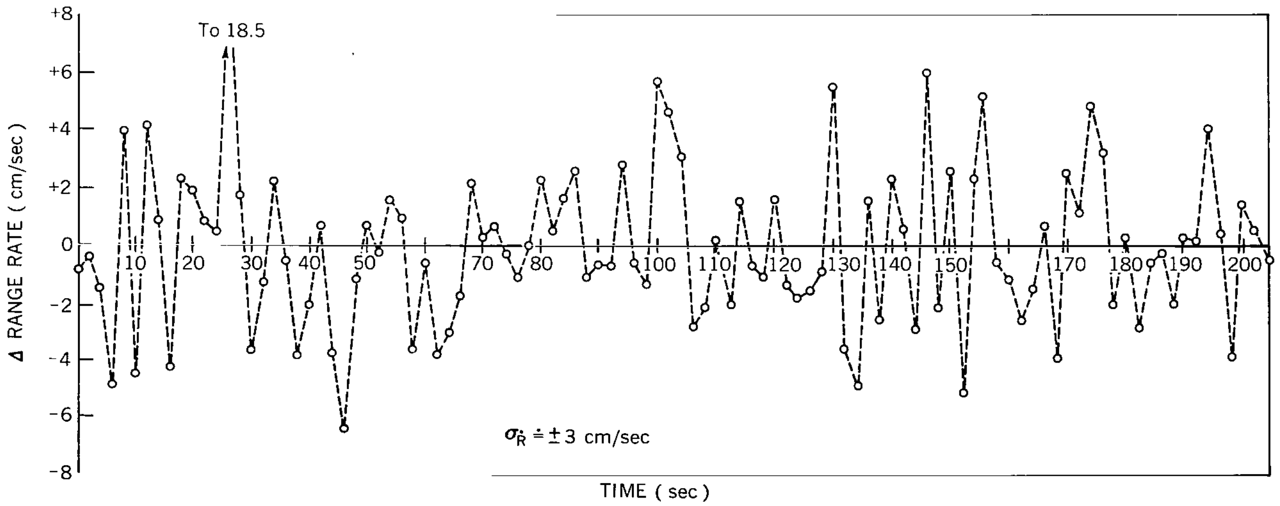

Figure 9—Data at 10,000 - 15,000 km (range rate).

Corresponding to Figures 8 and 9, the polynomials fitted to the data in the 10,000 to 15,000 km region are:

1. For range (R) data

$$T = t_{n-1} - t_o = 206 \text{ sec},$$

$$\Delta t = 2 \text{ sec},$$

$$k = \frac{t_k - t_o}{\Delta t} = 0, 1, 2, \cdots, 103,$$

$$R_k = 13408663.406 + 8937.701 k - 1.6413 k^2 \quad \text{(in meters)}, \tag{8}$$

$$\sigma_R = \pm 14.0 \text{ m} \quad \text{(See Equation 6 or 7)}.$$

2. For range rate ($\dot{R}$) data

$$T = 206 \text{ sec},$$

$$\Delta t = 2 \text{ sec},$$

$$k = \frac{t_k - t_o}{\Delta t} = 0, 1, 2, \cdots, 103,$$

$$\dot{R}_k = 4467.7189 - 1.6428 k + 0.0004 k^2 \quad \text{(in meters/sec)}, \tag{9}$$

$$\sigma_{\dot{R}} = \pm 0.031 \text{ m/sec}.$$

In the 10,000 to 15,000 km region, second degree polynomial fits to the range and range rate data over smaller values of T were also made. For all those intervals of T containing more than 15 data points, the second degree polynomial fitted the data best.

Range data in the region between 30,000 and 35,000 km must be divided into two classes:

1. Data extracted from a 4 kc ranging tone.
2. Data immediately following the above, which was extracted from a 20 kc ranging tone.

The plots of the residuals of the class 1 data are given in Figures 10 and 11 and those of class 2 data are given in Figures 12 and 13. Corresponding to the residual plots, the polynomials fitted to these data are:

1. For class 1 range data

$$T = 62 \text{ sec},$$

$$\Delta t = 2 \text{ sec},$$

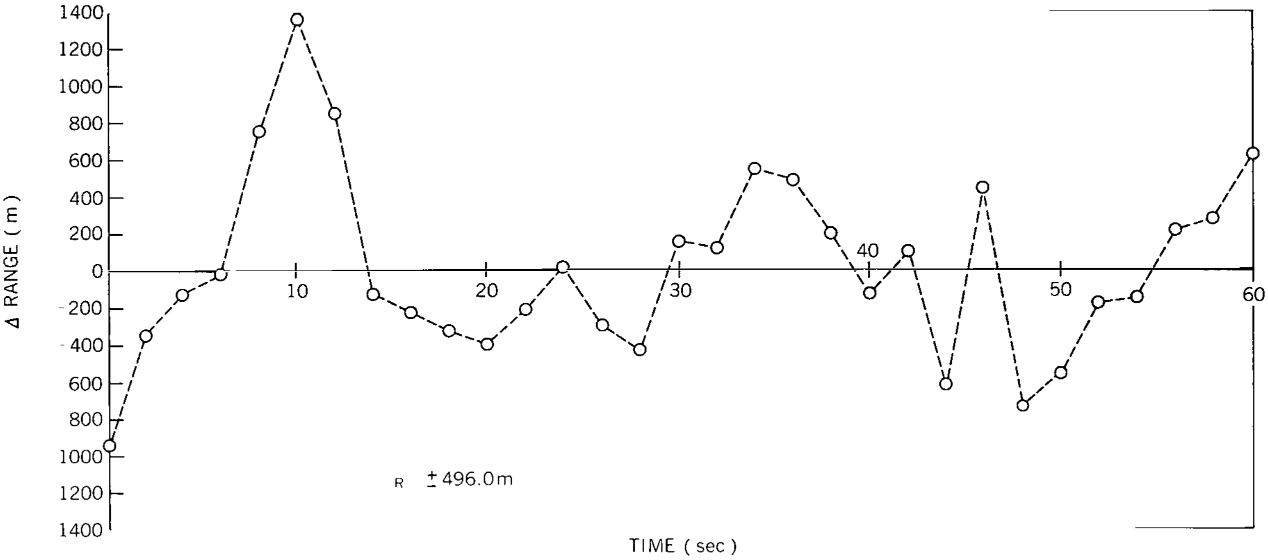

Figure 10—Data at 30,000 - 35,000 km (4 kc tone range, class 1).

$$k = \frac{t_k - t_o}{\Delta t} = 0,\ 1,\ 2,\ \cdots,\ 31\ ,$$

$$R_k = 34902312.641 + 1825.673\,k - 3.7109\,k^2 \quad \text{(in meters)}, \tag{10}$$

$$\sigma_R = \pm\ 496.0\ \text{m}\,.$$

2. For class 1 range rate data

$$T = 62\ \text{sec}\ ,$$

$$\Delta t = 2\ \text{sec}\ ,$$

$$k = \frac{t_k - t_o}{\Delta t} = 0,\ 1,\ 2,\ \cdots,\ 31\ ,$$

$$\dot{R}_k = 906.5779 - 0.4128\,k - 0.0002\,k^2 \quad \text{(in meters/sec)}, \tag{11}$$

$$\sigma_{\dot{R}} = \pm\ 0.0445\ \text{m/sec.}$$

3. For class 2 range data

$$T = 120\ \text{sec}\ ,$$

$$\Delta t = 1\ \text{sec}\ ,$$

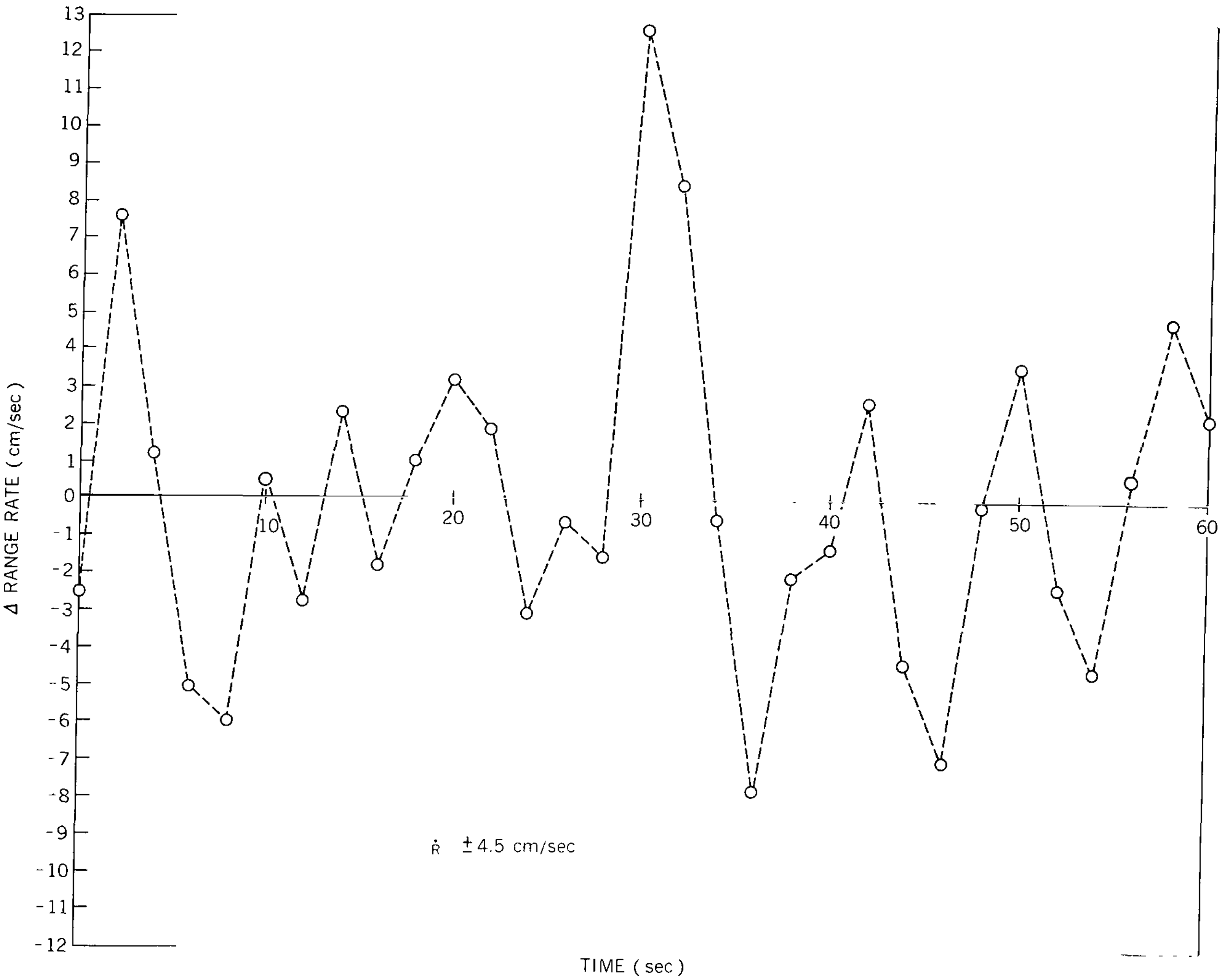

Figure 11—Data at 30,000 - 35,000 km (range rate, class 1).

$$k = \frac{t_k - t_o}{\Delta t} = 0,\ 1,\ 2,\ \cdots,\ 120,$$

$$R_k = 35647734.938 + 724.109\,k - 0.0992\,k^2 \quad \text{(in meters)}, \tag{12}$$

$$\sigma_R = \pm\ 19.1\ \text{m}.$$

4. For class 2 range rate data

$$T = 182\ \text{sec}\ ,$$

$$\Delta t = 1\ \text{sec}\ ,$$

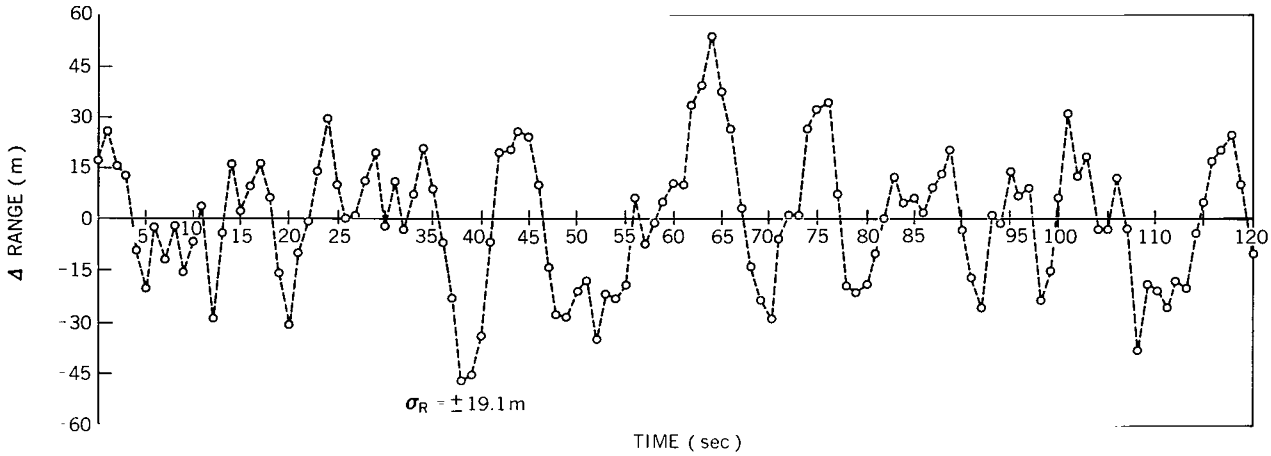

Figure 12—Data at 30,000 - 35,000 km (20 kc tone range, class 2).

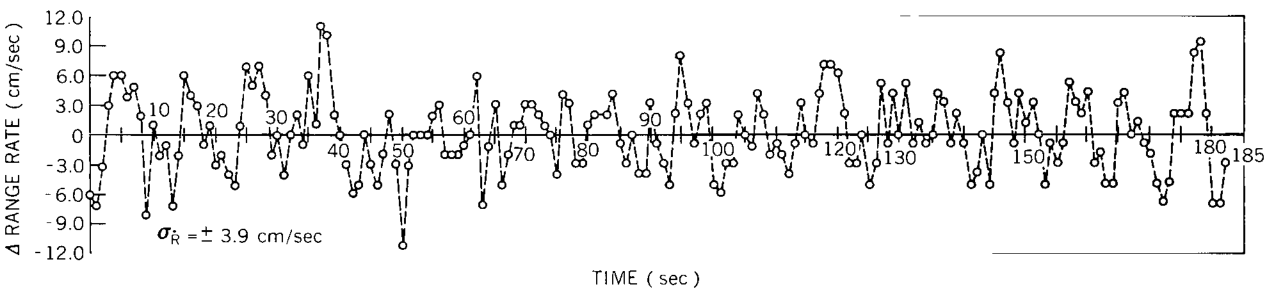

Figure 13—Data at 30,000 - 35,000 km (range rate, class 2).

$$k = \frac{t_k - t_o}{\Delta t} = 0,\ 1,\ 2,\ \cdots,\ 182,$$

$$\dot{R}_k = 725.674 - 0.1964k + 0.00001k^2 \quad \text{(in meters/sec)}, \tag{13}$$

$$\sigma_{\dot{R}} = \pm\ 0.039\ \text{m/sec}.$$

CONCLUSION

It should be noted that, from the standpoint of only the 100 or 20 kc highest range tone data, the precision of a signal measurement is better than ± 20 m in range and better than ± 0.05 m/sec in range rate for satellite heights between 10,000 and 35,000 km.

Although range measurements were not made with the 100 kc sidetone, the data proves the system's ability to make accurate (± 20 m) range measurements using the 20 kc sidetone.

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